Year 12 Physics ATAR

Special Relativity Practice Questions

Question 1

In the science fiction TV series Star Trek, the starship Enterprise is accelerated to a speed faster than the speed of light ("warp speed") for travel between star systems or galaxies. Explain why is it impossible to travel at or faster than the speed of light? (2)

According to Einstein's Special Theory of Relativity nothing can travel faster than the speed of light. This is indicated by time dilation which can increase a time period (e.g. tick of a clock) to infinity when travelling at the speed of light. This effectively means that time stops for the moving object viewed from a stationery reference frame.
From the point of view of the object (e.g the Enterprise) the spatial dimension of length in 3D space in the direction of travel contracts to zero. (Space is the object moving past the Enterprise)
At the speed of light the relativistic mass of any moving object increases to infinity. An impossible scenario that requires infinite energy and force to accelerate the mass any further. (any of the 3 acceptable)

Question 2

Two identical atomic clocks of extreme accuracy are synchronized at a space station in deep space. One of the clocks is put on a rocket and moves at a speed close to the speed of light relative to the space station.

(a) Explain how time is progressing on the moving clock as viewed from the space station. (1)

Time is progressing at a slower rate on the moving object as viewed from the stationery frame of reference. (Time between ticks is dilated)

(b) Explain the 'paradox' in this situation. (1)

Time is progressing at a slower rate on the space station as viewed from the clock. (The space station is moving relative to the rocket) – both are correct in their own frame of reference

Question 3

The starship Enterprise travels from Earth to Alpha Centauri (4.2 light-years away). One of the crew has an identical twin who remains on Earth. Ignore any acceleration of the Enterprise or gravity differences as this then requires consideration of the General Theory of Relativity which is beyond our course.

The Enterprise has a constant speed of 0.97*c* (*c* = speed of light). At this speed, how many years will it take to travel to Alpha Centauri as seen from Earth? (2)

$v_{av} = s / t \quad t = s / v_{av} = (4.2\ ly) / (0.97\ c)$

$t = 4.329896907$ years

The equation that governs length contraction is as follows: $l = l_0 \cdot \sqrt{1 - \frac{v^2}{c^2}}$

l = contracted length of an object moving at speed v, l_0 = length in a stationery reference frame

The equation that governs time dilation is as follows: $t_v = \frac{t_o}{\sqrt{1 - \frac{v^2}{c^2}}}$

t_0 = Proper time – the time elapsed for a an event to occur for an observer at rest relative to the event e.g the half-life of a radioactive substance, the time taken for the path between 2 locations in space (A and B) to move past a spaceship, the time between ticks on a clock that marks 1 second of time elapsed on that clock.

t = Dilated time - the time elapsed for the same event to occur for an object that is moving at speed v relative to an observer. e.g the half-life of the same substance within an object moving at speed v, the time taken for the spaceship to travel though space between locations A and B, the time between ticks on a clock that marks 1 second of time elapsed on that clock.

(c) At this speed what spatial dimension is contracting in the frame of reference of the Earth? (1)

The length of the Enterprise (in the direction of motion)

(d) At this speed what spatial dimension is contracting in the frame of reference of the Enterprise? (1)

The dimension of 3D space along the line of travel. The distance between Earth and Alpha Centauri is shorter in the frame of reference of Enterprise. Space is moving past the Enterprise

(e) Use the time dilation equation to calculate the time elapsed on the Enterprise as it travels between Earth and Alpha Centauri. Give your answer in years.

(3)

$$t_v = \frac{t_o}{\sqrt{1-\frac{v^2}{c^2}}} \qquad 4.32989690722 = \frac{t_o}{\sqrt{1-\frac{0.97^2}{1^2}}}$$

t_o = 1.05261922229 years passed by on the Enterprise as viewed from Enterprise (Event = Earth went backwards, Alpha Centauri arrived)
t_v = dilated time for the same event on Earth as viewed from Enterprise
Alternatively let t_0 = proper time for 1 second to pass on Earth clock
t_v= dilated time for 1 second to pass on moving Enterprise clock
t_v = 4.1134 Earth seconds for each second on the moving clock,
so years passed on Enterprise clock = 4.32989 / 4.1134 = 1.0526 years

(f) Use the length contraction equation and the equation $v_{av} = \frac{s}{t}$ to confirm the time taken to travel to Alpha Centauri from the perspective of the Enterprise crew. Give your answer in years.

(4)

$$l = l_0 \cdot \sqrt{1-\frac{v^2}{c^2}} \qquad l = 4.2\sqrt{1-\frac{0.97^2}{1^2}}$$

The distance between Earth and Alpha Centauri in the frame of reference of Enterprise as space moves past the Enterprise:
length = 1.02104064562 light years (the path length has contracted)

$v_{av} = s / t_0$ $\quad t_0 = s / v_{av}$ = (1.02104064562 ly) / (0.97 c)
t_0 = 1.05261922229 years passed by on the Enterprise as viewed from the Enterprise for the path through space to go past the Enterprise.

Note this is the proper time for this event to occur.
Note that the speed 0.97c is the speed of Space moving past the Enterprise from the reference frame of the Enterprise.

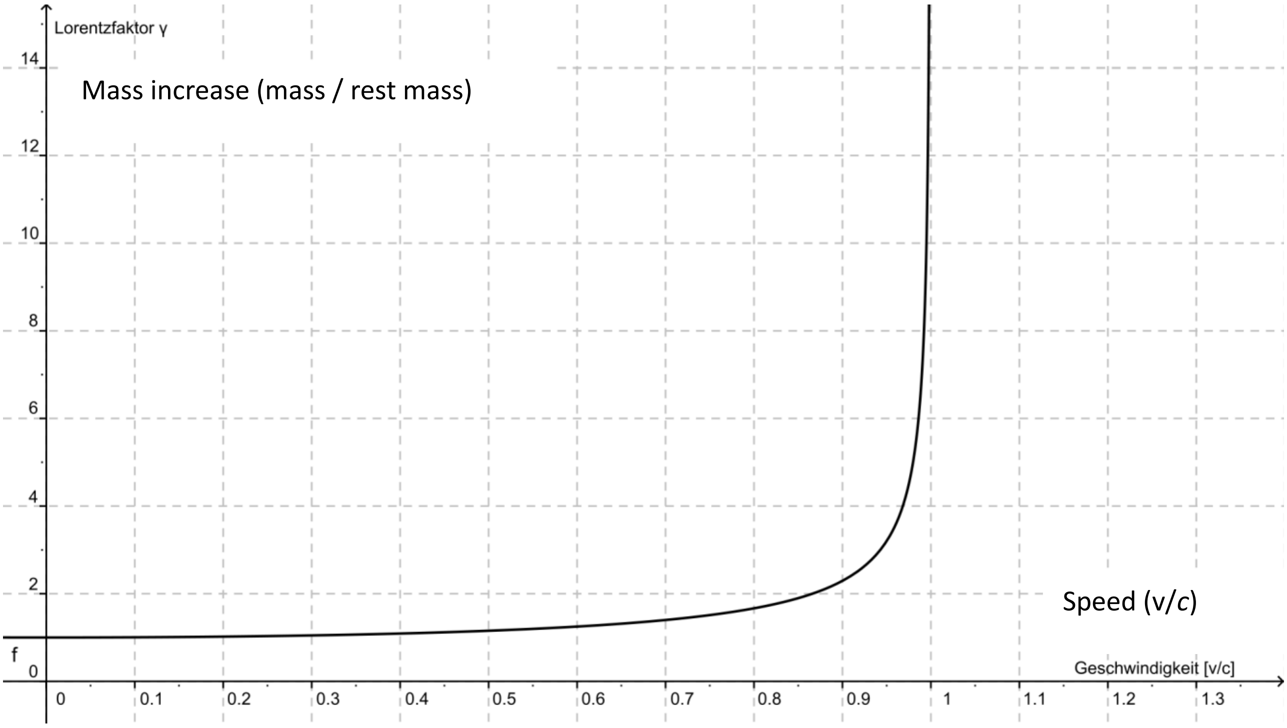
(g) What can you say about the relative age of the two twins when the Enterprise reaches Alpha Centauri?

(2)

Caution here as it's only truly explained by the General Theory when you can get both back into the same frame of reference. Accelerations are involved and SR does not deal with this.
The Earth twin argues that the Enterprise twin is aging at a slower rate
The Enterprise twin still argues that the Earth twin is getting younger according to SR as Earth is moving away from Enterprise at 0.97c
They are both correct in their own frame of reference.

Question 4

The following graph shows the factor by which mass increases with increasing velocity approaching the speed of light.



A proton of rest mass 1.67 x 10^{-27} kg is accelerated in the Large Hadron Collider until it reaches 0.95*c* (*c* = speed of light).

(a) Estimate the relativistic mass of the proton from the graph. (2)

From graph mass increases roughly by 3 × 1.67 × 10^{-27} × 3 = 5.01 × 10^{-27} kg

(b) What is the reason for this apparent increase in mass? (2)

According to Einstein's Theory of Special Relativity, the energy being put into the proton to accelerate it can be viewed as a mass increase.

Einstein derived the mathematical equation showing how mass changes with speed.

$$m_v = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Where m_0 = rest mass and m_v = relativistic mass at speed v (kg).

(c) Using the equation above, calculate the mass of the proton when it is moving at 0.99*c*. (2)

$$m_v = \frac{1.67\text{x}10^{-27}}{\sqrt{1 - \frac{(0.99c)^2}{c^2}}}$$ = 1.18 x 10^{-26} kg or about a 7 times increase.

(d) Why is it impossible for the proton to travel at or faster than the speed of light? (2)

The proton cannot be accelerated to the speed of light its mass approaches infinity, this would then require an infinitely large force and energy to accelerate the proton further.

Question 5

A freelance motoring journalist was caught speeding at 231 km h^{-1} during a test drive in a 110 km h^{-1} zone.

The $470,000 Ferrari California he was driving was seized by police under the WA hoon laws.

(a) A police car approaching the Ferrari from the opposite direction at 110 km h^{-1} recorded his speed on its radar. What speed was the Ferrari doing in the reference frame of the police car? (1)

341 km h^{-1}

(b) If the police car was following the Ferrari at 110 km h^{-1}, what speed would it register on the radar? (1)

121 km h^{-1}

(c) Is it necessary to take into account time and distance effects due to relativity in this question? Explain. (2)

No, relativistic effects only start to be noticeable near the speed of light (from the mass dilation graph only starts to be apparent at approx. 0.3c)

Question 6

The Andromeda galaxy is receding from Earth at about 0.3*c* (*c* = speed of light).

In the search for extraterrestrial life, a radio signal is sent from Earth into space towards Andromeda. At what speed is the radio signal measured from Andromeda? Explain your answer. (2)

$c = 3 \times 10^8$ m/s. According to Einstein's theory of special relativity the speed of light in a vacuum is constant no matter what the frame of reference.

Question 7

24 Global Positioning Satellites (GPS) at an altitude of 20 200 kilometres allow positions on Earth to be accurately determined to within 15 metres. About 10 satellites are visible to any ground-based receiver. Each satellite continually transmits messages which include the time the message was sent. The receiver measures the time of each message from the satellites and computes the distance to each satellite and then determines its position on the Earth's surface.

Over a period of exactly 24 hours the total time difference between the clock in the orbiting satellite and the clock in the GPS receiver is 3.80×10^{-5} s.

(a) According to the Special Theory of Relativity would the clocks on the moving satellites be running at a slower or faster rate as viewed from Earth? Explain.

(2)

According to Einstein's Theory of Special Relativity the moving clocks on the satellite run slower from Earth reference frame. (Time between ticks is longer).

(b) Which other theory of Einstein has a greater effect on the timing errors of the GPS system.

(1)

According to Einstein's Theory of General Relativity, time in the high gravity location (Earth), runs slower. Bigger effect than Special.

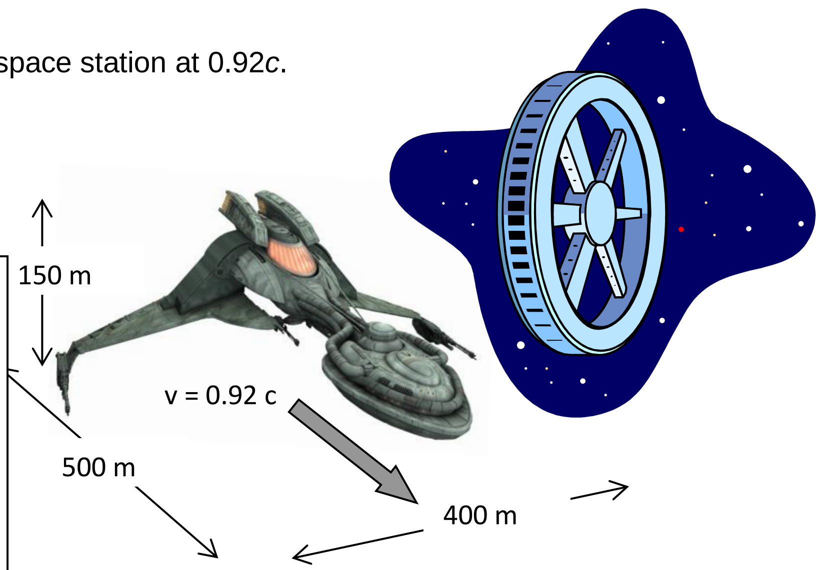
Question 8

A Klingon battleship passes by a Star Fleet space station at 0.92*c*.

a. What aspects of the Klingon ship's spatial dimensions are changed from the perspective of the space station?

The 500 m length is contracted (smaller)

All other dimensions are unchanged.



(2)

b. How does the Klingon warship perceive any changes in the 3D space it travels through compared to its view when at rest with the space station?

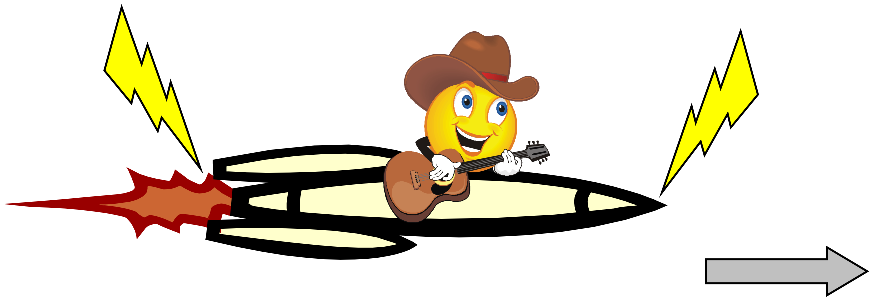
(2)

The distance along which it travels is shorter.

Question 9

Raymond the space cowboy is moving past you at 80% of the speed of light when from his point of view he sees lighting strike the front and back of his rocket at the same time. Do the lightning strikes occur simultaneously in your frame of reference? If yes explain why. If not explain which order you see the lightning strikes from your stationery frame of reference.

(3)



v = 0.80c

The back strike must have hit first. From a stationery reference you correctly predict it travels a longer path to catch up with the moving rocket and meet the second flash at Raymond's location. However, Raymond reasons that they travelled the same distance in the same time.

Question 10

Muons are leptons formed when cosmic radiation impacts air molecules 10 km to 60 km up in the Earth's atmosphere. The mean-life of a muon is 2.20 x 10^{-6} s. Muons travel at speeds of up to 0.999c and can penetrate deep into rock. They are detectable deep underground and underwater.

Rossi and Hall in 1940 measured muon impacts on a scintillation counter at an altitude of 3000 metres (568 counts h^{-1}) and at sea level (412 counts h^{-1}). From the perspective of Earth practically all muons should have decayed by the time they reach Earth if relativistic effects are not taken into account but only 3 mean-lives seem to have elapsed for them from an altitude of 60 km.

(a) What distance would muons be expected to travel in 3 of their mean-lives in their own reference frame

(2)

$3 \times 2.2 \times 10^{-6} \times 0.999 \times 3 \times 10^{8} = 1978$ m 1.978 km

Note – it is actually the path length of space travelling past the Muon.

(b) Explain why the Muons are able to survive a journey of this length at the speed they travel?

(3)

Special Relativity – length contraction. For an object moving through space its frame of reference says space is moving past it and the length dimension in space is contracted. In the frame of reference of the Muon they have only travelled a few km and there time passes normally. (Equation gives 2.68 km for this data)
Earth observers measure 60 km but with a time dilation for the Muon half-life.
Both are correct.